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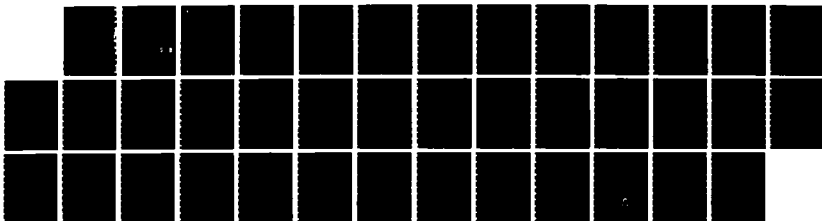
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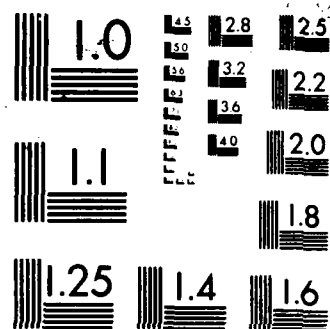
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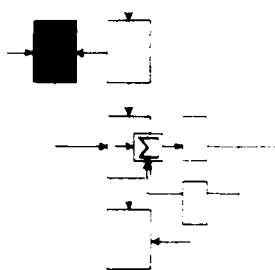
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NAVAL C³ DISTRIBUTED TACTICAL DECISIONMAKING

1. PROJECT OBJECTIVES

The objective of ~~the~~^{the} research is to address analytical and computational issues that arise in the modeling, analysis and design of distributed tactical decisionmaking. The research plan has been organized into two highly interrelated research areas:

- (a) Distributed Tactical Decision Processes; *and*
- (b) Distributed Organization Design.

The focus of the first area is the development of methodologies, models, theories and algorithms directed toward the derivation of superior tactical decision, coordination, and communication strategies of distributed agents in fixed organizational structures. The framework for this research is normative.

The focus of the second area is the development of a quantitative methodology for the evaluation and comparison of alternative organizational structures or architectures. The organizations considered consist of human decisionmakers with bounded rationality who are supported by C³_D systems. The organizations function in a hostile environment where the tempo of operations is fast; consequently, the organizations must be able to respond to events in a timely manner. The framework for this research is descriptive. *Key -*

2. STATEMENT OF WORK

The research program has been organized into seven technical tasks - four that address primarily the theme of distributed tactical decision processes and three that address the design of distributed organizations. An eighth task addresses the integration of the results. They are:



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2.1 Real Time Situation Assessment

Static hypothesis testing, the effect of human constraints and the impact of asynchronous processing on situation assessment tasks will be explored.

2.2 Real Time Resource Allocation

Specific research topics include the use of algebraic structures for distributed decision problems, aggregate solution techniques and coordination.

2.3 Impact of Informational Discrepancy

The effect on distributed decisionmaking of different tactical information being available to different decisionmakers will be explored. The development of an agent model, the modeling of disagreement, and the formulation of coordination strategies to minimize disagreement are specific research issues within this task.

2.4 Constrained Distributed Problem Solving

The agent model will be extended to reflect human decisionmaking limitations such as specialization, limited decision authority, and limited local computational resources. Goal decomposition models will be introduced to derive local agent optimization criteria. This research will be focused on the formulation of optimization problems and their solution.

2.5 Evaluation of Alternative Organizational Architectures

This task will address analytical and computational issues that arise in the construction of the generalized performance-workload locus. This locus is used to describe the performance characteristics of a decisionmaking organization and the workload of individual decisionmakers.

2.6 Asynchronous Protocols

The use of asynchronous protocols in improving the timeliness of the organization's response is the main objective of this task. The tradeoff between timeliness and other performance measures will be investigated.

2.7 Information Support Structures

In this task, the effect of the C³ system on organizational performance and on the decisionmaker's workload will be studied.

2.8 Integration of Results

A final, eighth task, is included in which the various analytical and computational results will be interpreted in the context of organizational bounded rationality.

3. STATUS REPORT

In the context of the first seven tasks outlined in Section 2, a number of specific research problems have been formulated and are being addressed by graduate research assistants under the supervision of project faculty and staff. Research problems which were completed prior to or were not active during this last quarter have not been included in the report.

3.1 DISTRIBUTED TEAM HYPOTHESIS TESTING WITH SELECTIVE COMMUNICATIONS

Background: In Command-and-Control-and-Communications (C³) systems multiple hypothesis-testing problems abound in the surveillance area. Targets must be detected and their attributes must be established: this involves target discrimination and identification. Some target attributes, such as location, are best observed by sensors such as radar. More uncertain target locations are obtained by passive sensors, such as sonar or IR sensors. However, target identity information requires other types of sensors (such as ESM

receivers, IR signature analysis, human intelligence etc). As a consequence in order to accurately locate and identify a specific target out of possibly large potential population (including false targets) one must design a detection and discrimination system which involves the fusing of information from several different sensors generating possibly specialized information about the target. These sensors may be allocated on a platform (say a ship in a Naval battle group) and be physically dispersed as well (ESM receivers exist in every ship, aircraft, and submarine). The communication of information among this diverse sensor family may be difficult (because of EMCON restrictions) and is vulnerable to enemy countermeasure actions (physical destruction and jamming). It is this class of problems that motivates our research agenda.

Problem Statement: We are conducting research on distributed multiple hypothesis testing using several decisionmakers, and teams of decisionmakers, with distinct private information and limited communications. The goal of this research is to unify our previous research in situation assessment, distributed hypothesis testing, and impact of informational discrepancy; and to extend the methodology, mathematical theory and computational algorithms so that we can synthesize and study more complex organizational structures. The solution of this class of basic research problems will have impact in structuring the distributed architectures necessary for the detection, discrimination, identification and classification of attributes of several targets (or events) by a collection of distinct sensors (or dispersed human observers).

The objective of the distributed organization will be the resolution of several possible hypotheses based on many uncertain measurements. Each hypothesis will be characterized by several attributes. Each attribute will have a different degree of observability to different decisionmakers or teams of decisionmakers; in this manner, we shall model different phenomena. Since each hypothesis will have several attributes, it follows that in order to reliably confirm or reject a particular hypothesis, two or more decisionmakers (or two or more teams of decisionmakers) will have to pool and fuse their knowledge.

Extensive and unnecessary communication among the decisionmakers will be discouraged by explicitly assigning costs to certain types of communication. In this manner, we shall seek to understand and isolate which communications are truly vital in the organizational performance; the very problem formulation will discourage communications whose impact upon performance is minimal. Quantitative tradeoffs will be sought.

Another feature which will be incorporated relates to the vulnerability of the distributed decision process to enemy countermeasures. Thus, in our distributed decision models we shall assume that there is a finite probability that the actions (decision and/or conclusions of any one particular decisionmaker will be distorted or destroyed due to enemy action. As a consequence, the organization of the decision teams, the protocols, and the decision rules must explicitly take into account the vulnerability issue. As a minimum, a certain level of decisionmaking redundancy must exist in the distributed organization; the coordination strategies and the protocols that isolate "damaged" decisionmakers will be developed. We shall seek to determine, in a quantitative setting, the minimum required level of decisionmaker redundancy as a function of the degree of vulnerability to enemy countermeasures (such as jamming).

We stress that we shall strive to design distributed organizational architectures in which teams of teams of decisionmakers interact. For example, a team may consist of a primary decisionmaker together with a consulting decisionmaker — the paradigm used by Papastavrou and Athans.

The methodology that we plan to employ will be mathematical in nature. To the extent possible we shall formulate the problems as mathematical optimization problems. Thus, we seek normative solution concepts. To the extent that human bounded rationality constraints are available, these will be incorporated in the mathematical problem formulation. In this case, the nature of the results will correspond to what is commonly referred to as normative/descriptive solutions. Therefore, we visualize a dual benefit of our basic research results. From a purely mathematical point of view, the

research will yield nontrivial advances to the distributed hypothesis-testing problem; an extraordinary difficult problem from a mathematical point of view. From a psychological perspective, we hope that the normative results will suggest counterintuitive behavioral patterns of -- even perfectly rational -- decisionmakers operating in a distributed tactical decisionmaking environment; these will set the stage for designing empirical studies and experiments and point to key variables that should be observed, recorded and analyzed by cognitive scientists. From a military C³ viewpoint, the results will be useful in structuring distributed architectures for the surveillance function.

Progress to Date: Research was initiated in September 1987. At present we are in the modeling and problem formulation phase. The challenge is to pose the problem in such a way so that its generic richness is preserved, yet having a chance for mathematical solutions which will provide insight.

We have developed a simple model for capturing the effects of countermeasures. Suppose that we have a decisionmaker that makes a binary decision, i.e., YES, I believe that I see a target vs. NO, I do not believe that a target is there. We can have a small but finite probability that when the decisionmaker meant to say YES the other team members hear NO, and vice versa. The degree of the countermeasures intensity can be quantified by the numerical values of the assigned probability. This way of modeling the impact of enemy countermeasures does not complicate the mathematics very much in the distributed hypothesis-testing algorithms.

Many more mathematical models and tentative approaches will have to be developed before we can start our optimization studies. This research will most probably form the core of the Ph.D. research of J. Papastavrou under the supervision of Professor M. Athans.

Documentation: None as yet.

3.2 DISTRIBUTED HYPOTHESIS TESTING WITH MANY AGENTS

Background: The goal of this research project is to develop a better understanding of the nature of the optimal messages to be transmitted to a central command station (or fusion center) by a set of agents who receive different information on their environment. In particular, we are interested in solutions of this problem which are tractable from the computational point of view. Progress in this direction has been made by studying the case of a large number of agents. Normative/prescriptive solutions are sought.

Problem Statement: Let H_0 and H_1 be two alternative hypotheses on the state of the environment and let there be N agents (sensors) who possess some stochastic information related to the state of the environment. In particular, we assume that each agent i observes a random variable y_i with known conditional distribution $P(y_i | H_j)$, $j = 0, 1$, given either hypothesis. We assume that all agents have information of the same quality, that is, the random variables are identically distributed. Each agent transmits a binary message to a central fusion center, based on his information y_i . The fusion center then takes into account all messages it has received to declare hypothesis H_0 or H_1 true. The problem consists of determining the optimal strategies of the agents as far as their choice of message is concerned. This problem has been long recognized as a prototype problem in team decision theory: it is simple enough so that analysis may be feasible, but also rich enough to allow nontrivial insights into optimal team decision making under uncertainty.

Results: This problem is being studied by Prof. J. Tsitsiklis. Under the assumption that the random variables y_i are conditionally independent (given either hypothesis), it is known that each agent should choose his message based on a likelihood ratio test. Nevertheless, we have constructed examples which show that even though there is perfect symmetry in the problem, it is optimal to have different agents use different thresholds in their likelihood ratio tests. This is an unfortunate situation, because it severely complicates the numerical solution of the problem (that is, the explicit

computation of the threshold of each agent). Still, we have shown that in the limit, as the number of agents becomes large, it is asymptotically optimal to have each agent use the same threshold. Furthermore, there is a simple effective computational procedure for evaluating this single optimal threshold.

More recently, we have extended our results in several directions.

We have shown that if each agent is to transmit K-valued, as opposed to binary messages, then still each agent should use the same decision rule, when the number of agents is large. Unfortunately, however, the computation of this particular decision rule becomes increasingly broader as K increases.

We have also investigated the case of M-ary ($M > 2$) hypothesis testing and constructed examples showing that it is better to have different agents use different decision rules, even in the limit as $N \rightarrow \infty$. Nevertheless, we have shown that the optimal set of decision rules is not completely arbitrary. In particular, it is optimal to partition the set of agents into at most $M(M-1)/2$ groups and, for each group, each agent should use the same decision rule. The decision rule corresponding to each group and the proportion of the agents assigned to each group may be determined by solving a linear programming problem, at least in the case where the set of possible observations by each agent is finite.

Documentation:

- [1] J. Tsitsiklis, "On Threshold Rules in Decentralized Detection," Proc. of the 25th IEEE Conference on Decision and Control, Athens, Greece, December 1986; also Report LIDS-P-1570, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, June 1986.

3.3 COMMUNICATION REQUIREMENTS OF DIVISIONALIZED ORGANIZATIONS

Background: In typical organizations, the overall performance cannot be evaluated simply in terms of the performance of each subdivision, as there

may be nontrivial coupling effects between distinct subdivisions. These couplings have to be taken explicitly into account; one way of doing so is to assign to the decision maker associated with the operation of each division a cost function which reflects the coupling of his own division with the remaining divisions. Still, there is some freedom in such a procedure: For any two divisions A and B it may be the responsibility of either decision maker A or decision maker B to ensure that the interaction does not deteriorate the performance of the organization. Of course, the decision maker in charge of those interactions needs to be informed about the actions of the other decision maker. This leads to the following problem. Given a divisionalized organization and an associated organizational cost function, assign cost functions to each division of the organization so that the following two goals are met: a) the costs due to the interaction between different divisions are fully accounted for by the subcosts of each division; b) the communication interface requirements between different divisions are small. In order to assess the communication requirements of a particular assignment of costs to divisions, we take the view that the decision makers may be modeled as boundedly rational individuals, that their decision making process consists of a sequence of adjustments of their decisions in a direction of decreasing costs, while exchanging their tentative decisions with other decision makers who have an interest in those decisions. We then require that there are enough communications so that this iterative process converges to an organizationally optimal set of decisions.

Problem Statement: Consider an organization with N divisions and an associated cost function $J(x_1, \dots, x_N)$, where x_i is the set of decisions taken at the i -th division. Alternatively, x_i may be viewed as the mode of operation of the i -th division. The objective is to have the organization operating at set of decisions (x_1, \dots, x_N) which are globally optimal, in the sense that they minimize the organizational cost J . We associate with each division a decision maker DM_i , who is in charge of adjusting the decision unables x_i . We model the decision makers as "boundedly rational" individuals; mathematically, this is translated to the assumption that each decision maker will slowly and iteratively adjust his decisions in a

direction which reduces the organizational costs. Furthermore, each decision maker does so based only on partial knowledge of the organizational cost, together with messages received from other decision makers.

Consider a partition $J(x_1, \dots, x_N) = \sum_{i=1}^N J^i(x_1, \dots, x_N)$ of the organizational cost. Each subcost J^i reflects the cost incurred to the i -th division and in principle should depend primarily on x_i and only on a few of the remaining x_j 's. We then postulate that the decision makers adjust their decisions by means of the following process (algorithm):

- (a) DM_i keeps a vector x with his estimates of the current decisions x_k of the other decision makers; also a vector λ with estimates of $\lambda_i^k = \partial J^k / \partial x_i$, for $k \neq i$. (Notice that this partial derivative may be interpreted as DM_i 's perception of how his decisions affect the costs incurred to the other divisions.
- (b) Once in a while DM_i updates his decision using the rule $x_i := x_i - \gamma \sum_{k=1}^N \lambda_i^k$. (γ is a small positive scalar) which is just the usual gradient algorithm.
- (c) Once in a while DM_i transmits his current decision to other decision makers.
- (d) Other decision makers reply to DM_i , by sending a updated value of the partial derivative $\partial J^k / \partial x_i$.

It is not hard to see that for the above procedure to work it is not necessary that all DM's communicate to each other. In particular, if the subcost J^i depends only on x_i , for each i , there would be no need for any communication whatsoever. The required communications are in fact determined by the sparsity structure of the Hessian matrix of the subcost functions J^i . Recall now that all that is given is the original cost function J ; we therefore have freedom in choosing the J^i 's and we should be able to do this in a way that introduces minimal communication requirements; that is, we want to minimize the number of pairs of decision makers who need to communicate to each other.

The above problem is a prototype organizational design problem and we expect that it will lead to reasonable insights in good organizational structures. On the technical side, it may involve techniques and tools from graph theory. Once the above problem is understood and solved, the next step is to analyze communication requirements quantitatively. In particular, a distributed gradient algorithm such as the one introduced above converges only if the communication (between pairs of DM's who need to communicate) is frequent enough. We will then investigate the required frequencies of communication as a function of the strength of coupling between different divisions.

Progress to Date: A graduate student, C. Lee, supervised by Prof. J. Tsitsiklis, has undertaken the task of formulating the problem of finding partitions that minimize the number of pairs of DM's who need to communicate to each other as the topic of his SM research. The literature search phase has been completed, and different problem formulations are being investigated. It was realized that with a naive formulation the optimal allocation of responsibilities, imposing minimal communication requirements, corresponds to the centralization of authority. Thus, in order to obtain more realistic and meaningful problems we are incorporating a constraint requiring that no agent should be overloaded. Certain preliminary results have been already obtained for a class of combinatorial problems, corresponding to special cases of the problem of optimal organizational design, under limited communications.

Documentation: None as yet.

3.4 COMMUNICATION COMPLEXITY OF DISTRIBUTED CONVEX OPTIMIZATION

Background: The objective of this research effort is to quantify the minimal amount of information that has to be exchanged in an organization, subject to the requirement that a certain goal is accomplished, such as the minimization of an organizational cost function. This problem becomes interesting and relevant under the assumption that no member of the organization "knows" the entire function being minimized, but rather each agent has knowledge of only

a piece of the cost function. A normative/prescriptive solution is sought.

Problem Formulation: Let f and g be convex function of n variables. Suppose that each one of two agents (or decisionmakers) knows the function f (respectively g), in the sense that he is able to compute instantly any quantities associated with this function. The two agents are to exchange a number of binary messages until they are able to determine a point x such that $f(x) + g(x)$ comes within ϵ of the minimum of $f + g$, where ϵ is some prespecified accuracy. The objective is to determine the minimum number of such messages that have to be exchanged, as a function of ϵ and to determine communication protocols which use no more messages than the minimum amount required.

Results: The problem is being studied by Professor John Tsitsiklis and a graduate student, Zhi-Quan Luo. We have shown that at least $O(n \log 1/\epsilon)$ messages are needed and a suitable approximate and distributed implementation of ellipsoid-type algorithms work with $O(n^2 \log^2 1/\epsilon)$ messages. The challenge is to close this gap. This has been accomplished for the case of one-dimensional problems $n = 1$, for which it has been shown that $O(\log 1/\epsilon)$ messages are also sufficient. More recently, we have succeeded in generalizing the technique employed in the one-dimensional case, and we obtained an algorithm with $O(n^2 \log 1/\epsilon)$ communications; we thus have an algorithm which is optimal, as far as the dependence of ϵ is concerned. The question of the dependence of the amount of communications on the dimension of the problem ($O(n)$ versus $O(n^2)$) seems to be a lot harder and, at present, there are no available techniques for handling it.

An interesting qualitative feature of the communication-optimal algorithms discovered thus far is the following: It is optimal to transmit aggregate information (the most significant bits of the gradient of the function optimized) in the beginning; then, as the optimum is approached more refined information should be transferred. This very intuitive result seems to correspond to realistic situations in human decisionmaking.

Another problem which is currently being investigated concerns the case where there are $K > 2$ decision makers cooperating for the minimization of $f_1 + \dots + f_k$ where each f_i is again a convex function.

Documentation:

- [1] J. N. Tsitsiklis and Z.-Q. Luo, "Communication Complexity of Convex Optimization," Report LIDS-P-1617, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, October 1986; also, Proceedings 25th IEEE Conference on Decision and Control, Athens, Greece, December 1986.

3.5 DESIGN AND EVALUATION OF ALTERNATIVE ORGANIZATIONAL ARCHITECTURES

Background: The bounded rationality of human decisionmakers and the complexities of the tasks they must perform mandate the formation of organizations. Organizational architectures distribute the decisionmaking workload among the members; different architectures impose different individual loads, lead to different organizational bounded rationality, and result in different organizational performance. Two performance measures have been investigated up to now: accuracy and time delay. An approach to the evaluation and comparison of alternative organizational architectures, that provides insight into the effect structure has on organizational bounded rationality, is the use of a generalized performance-workload locus.

Problem Statement: The development of design guidelines for distributed organizational architectures is the objective. To achieve this objective, a sequence of steps has been defined. Each step in the sequence requires the solution of both modeling and computational problems:

- (1) Development of efficient computational procedures for constructing the generalized performance-workload locus.
- (2) Analysis of the functional relationship between internal decision strategies and workload (i.e., the properties of the mapping from strategy space to workload space).
- (3) Development of quantitative and qualitative relationships between

organizational architecture and the geometry of the performance-workload locus.

Remarks: The work implied in the problem statement requires modeling, analysis, and computation. The use of computer graphics is an integral part of the computational procedures.

At the beginning of this reporting period, the direction of research changed. With the basic tools for the computation of organizational performance developed and implemented, the emphasis has been shifted to formulating the organizational design problem. This task has been divided into two subtasks that correspond to two thesis projects.

(i) Generation of Organizational Structures

Background: Most of the theoretical developments in decision and control theory have addressed the problem of analyzing the performance of a given organizational form. In this case, the organizational structure is fixed and well defined. Changes in the topology of the organization may be made to improve performance, but they always remain incremental. There is need to develop a methodology for generating feasible organizational forms.

Problem Statement: Develop a mathematical framework for generating organizational forms that satisfy some structural and some application specific constraints.

Results: This problem has been addressed by P. Remy under the supervision of Dr. A. H. Levis. The first step in the procedure was the definition of the Petri Net and the corresponding data structure for the interacting decisionmaker. In the past, information sharing was allowed only between the situation assessment stage and the information fusion process. This assumption has been relaxed to allow four different forms of information sharing - each form depends on the source of the information (e.g., is one DM informing the other of his situation assessment or of his response?) and on the destination. For example, the situation assessment of one DM may be the

input to the next one in a serial or hierarchical organization. After defining the set of possible interactions, a combinatorial problem could be formulated. The dimensionality of this problem is prohibitive, if no constraints on the structure are imposed. There are $2^{2n(2n-1)}$ organizational forms in this formulation, where n is the number of decisionmakers. These organizational forms are called Well Defined Nets (WDNs) of dimension n . An algorithmic approach has been developed that reduces the problem to a computationally tractable one.

A series of propositions, proved by Remy, set the theoretical basis of the algorithm. These propositions constitute significant extensions of Petri Net Theory. The first proposition establishes that if the source and the sink places of a Petri Net representing a WDN are combined into a single place and if the resulting Petri Net is strongly connected, then it is an event graph (a special class of Petri Nets).

Then, two sets of constraints are introduced to eliminate unrealistic organizational forms. The first set, structural constraints, define what kinds of interactions between decisionmakers must be ruled out. User-defined constraints allow the designer to introduce specific structural characteristics that are appropriate (or are mandated) for the particular design problem.

The first structural constraint imposes a minimum degree of connectivity in the organization; it eliminates structures that do not represent a single integrated organization and ensures that the flow of information is continuous. The second constraint allows acyclical organizations only. This restriction is made to avoid deadlock and the circulation of messages. The third constraint prohibits one decisionmaker from sending the same data to different stages of another decisionmaker's model. This is a technical, model-specific restriction that recognizes the fact that the stages of decisionmaking are a modeling artifice that should not introduce extraneous complexity. The last constraint restricts the situation assessment stage to receiving a single input; multiple inputs can be received at the information fusion stage.

The user-defined constraints are arbitrary; they reduce the degrees of freedom in the design process. A WDN that satisfies the user-defined constraints is called an Admissible Organizational Form. An admissible form that also satisfies the structural constraints is a Feasible Organization.

The second proposition characterizes formally the admissible organizational forms as subsets of the set of WDNs. Furthermore, it introduces the concept of maximal and minimal elements of the sets. A maximal element of the set of Feasible Organizations is called a Maximally Connected Organization (MAXO) while a minimal one is called a Minimally Connected Organization (MINO).

The third proposition establishes that any feasible organization is bounded from above by at least one MAXO and from below by at least one MINO.

With this characterization of the feasible structures, what remains is to develop a procedure for generating them. The procedure is based on the concept of simple paths developed by Jin (or the s-invariants of Petri Net theory). The fourth and fifth propositions lead to the algorithm for generating feasible organizations. They show that one can construct the set of all the possible unions of simple paths. Then one can determine all the MAXOs and the MINOs of the set. These MAXOs and MINOs bound the solution set. Any feasible organization form is a subset of a MAXO and has one or more MINOs as subsets. By adding simple paths to every MINO until a MAXO is reached, one can construct the complete set of Feasible Organizations.

This is a powerful result, both theoretically and computationally, that opens the way for generating classes of feasible organizational forms that meet, a priori, some structural and performance requirements. The partial ordering of the solutions (another result established by Remy) allows the use of lattice theory to analyze the properties of various architectures.

Documentation:

- [1] P. A. Remy, "On The Generation of Organizational Architectures Using Petri Nets," LIDS-TH-1630 S.M. Thesis, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, December 1986.
- [2] P. A. Remy, A. H. Levis, V.Y.-Y. Jin, "On The Design of Distributed Organizational Structures," Proc. 10th IFAC World Congress, Munich, FRG, July 1987.
- [3] P. A. Remy and A. H. Levis, "On the Generation of Organizational Architectures Using Petri Nets," LIDS-P-1634, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, December 1986.

(ii) Design of Organizations

Objective: Given a feasible organizational architecture, develop a methodology for (a) identifying the functions that must be performed by the organization in order that the task be accomplished, (b) selecting the resources (human, hardware, software) that are required to implement these functions, and (c) integrating these resources - through interactions - so that the system operates effectively.

Progress to Date: This research problem is being investigated by Stamos K. Andreadakis under the supervision of Dr. A. H. Levis. The proposed design methodology consists of two stages.

In the first stage, the specific objective is to meet the requirements for the two measures of performance - accuracy and timeliness. This is accomplished by selecting the functions that are to be performed by the organization in support of the task. The emphasis in this stage is on the design of the protocols that specify the interactions between the processes that instantiate the functions.

In the second stage, the objective is allocating the various functions to different decisionmakers so that the individual workload constraints are met. The allocation must satisfy additional considerations such as the need for

some redundancy so that the system has high degrees of survivability and reconfigurability.

The two stages of the iterative design process have been specified and the corresponding algorithms have been designed. The algorithms are being implemented on the design workstation.

Documentation: None as yet. A thesis proposal by S. K. Andreadakis for a doctoral dissertation based on this work has been accepted. An invited paper for the 10th IFAC World Congress is in preparation.

3.6 ASYNCRHONOUS PROTOCOLS

Background: In distributed tactical decisionmaking organizations (DTDMO) supported by C^3 systems, timeliness is of critical importance. The ability of an organization to carry out tasks in a timely manner is indeed a determinant factor of effectiveness. There are two types of constraints which affect the time-performance of a DTDMO. The first type is related to the internal organizational structure that determines how the various operations occur in the process: some tasks processed sequentially, while others are processed concurrently. The sequential and concurrent events are coordinated by the communication and execution protocols among the individual organization members. The second type of constraints consist of time and resource constraints. The time constraints derive from the task execution times -- the time necessary to perform each task. The organization also has limited resources; depending on which of the resources are available at a given instant, some activities can take place while others must be delayed.

The Petri Net formalism provides a convenient tool for analyzing the behavior of organizations with asynchronous protocols that allow for concurrent processing.

Problem Statement: In earlier work by Jin, the response time of a decisionmaking organization was computed using an algorithm based on the Petri Net representation. The definition of response time was the time interval between the moment a stimulus is received by the organization and the moment a response is made. This measure of performance is a static measure insofar that it assumes that there are no other tasks being processed by the organization. A more realistic estimate of response time will be obtained, if the dynamic behavior of the organization is taken into account. More precisely, the research problem is to evaluate the performance of a DTDMO with respect to the following time-related measures:

- (a) Maximum Throughput Rate: This is the maximum rate at which external inputs can be processed; a higher rate would lead to the formation of queues of unbounded length.
- (b) Execution Schedule: Let processing of arriving inputs start at t_0 and let the inputs be processed at the maximum throughput rate. The earliest instants of time at which the various tasks can be performed in the repetitive process constitute the optimum execution schedule; any other schedule will lead to longer response times.

Results: The time-related performance of a DTDMO, as measured by the maximum throughput rate and the execution schedule, has been analyzed and evaluated. The approach was based on modeling the DTDMO as a Timed Petri Net. Two constraints have been modeled to characterize the bounded rationality of human decisionmakers. The time associated with individual processes reflects a processing rate limitation, while the resource limitation models the limited capacity of short-term memory, which bounds the amount of information that a DM can handle at the same time. Both considerations are modeled as a constraint on the total number of inputs that can be processed simultaneously.

The maximum throughput rate has been expressed as a function of the resource and time constraints in the following manner: The inclusion of the resource constraints in the Petri Net model results in directed circuits (or loops) which are characterized by: (a) the circuit processing time, μ , defined as the sum of the different task processing times of the circuit. μ represents the amount of time it takes one input to complete the processing operations of the circuit; (b) the resources available, n , which bound the total number of inputs that can be processed at the same time in the circuit.

For a given circuit, the ratio n/μ characterizes the average circuit processing rate. The minimum average circuit processing rate, taken over all the directed circuits of the net, determines the maximum throughput rate of the deterministic systems, i.e., when all the task processing times are deterministic. For the case of stochastic processing times, an upper bound is obtained for the maximum throughput rate. In that case, the average circuit processing time can be computed. The determination of the critical circuits, for which the corresponding average processing rate is minimal, provides a clear way of comparing different organizations. These critical circuits are the ones that, because of the time and resource constraints, bound the throughput rate. Therefore, there is now a direct way to identify how different constraints affect organizational performance. Consequently, the problem of modifying the right constraints so as to improve the performance of the organizations (and meet mission requirements) becomes transparent.

A method for obtaining and analyzing the exact execution schedule when processing times are deterministic has been developed. A representation, defined by the slices of the Petri Net, allows for the precise characterization of the causal relations in the DTDMO. The causality relationships result in the partial ordering of the different operations. The execution schedule so obtained determines the earliest instants at which the various tasks can be executed in real-time for a process that occurs repetitively.

The contribution of this research, carried out by H. P. Hillion under the supervision of Dr. A. H. Levis, is that it develops two MOPs that characterize the time-related behavior of a distributed tactical decisionmaking organization. Furthermore, the concepts and algorithms developed are oriented toward design: they indicate which design parameters need to be changed to meet requirements.

Documentation:

- [1] H. P. Hillion, "Performance Evaluation of Decisionmaking Organizations Using Timed Petri Nets," LIDS-TH-1590, S.M. Thesis, Laboratory for Information and Decision Systems, MIT, August 1986.
- [2] H. P. Hillion and A. H. Levis, "Timed Event-Graph and Performance Evaluation of Systems," LIDS-P-1639, Laboratory for Information and Decision Systems, MIT, January 1987.

4. OTHER ACTIVITIES

4.1 Awards

The IEEE Control Systems Society has conferred the OUTSTANDING PAPER AWARD for a paper published in the IEEE Transactions on Automatic Control during the years 1984-1985 to Professors John N. Tsitsiklis and Michael Athans for their paper "On the Complexity of Decentralized Decision Making and Detection Problems" which was published in the May 1985 issue of the IEEE Transactions on Automatic Control.

The award was presented to the authors at the 25th IEEE Conference on Decision and Control, Athens, Greece on December 10, 1986.

This paper was based on work carried out under the DTDM program.

4.2 Meetings

A joint meeting was held on December 22, 1986 to discuss experimental programs in distributed tactical decisionmaking organizations with the member of the research team from ALPHATECH, Inc., and the faculty and students from the University of Connecticut. The meeting was informative and productive. It was agreed to continue such meetings on a bimonthly basis. The next meeting will be in February at the University of Connecticut and its focus will be the modeling of resource management problems.

5. RESEARCH PERSONNEL

| | |
|-------------------------|---------------------------|
| Prof. Michael Athans, | Co-principal investigator |
| Dr. Alexander H. Levis, | Co-principal investigator |
| Prof. John Tsitsiklis | |
| Dr. Jeff Casey | |

| | |
|---------------------------|-------------------------------------|
| Mr. Stamatios Andreadakis | graduate research assistant (Ph.D.) |
| Ms. Victoria Jin | graduate research assistant (Ph.D.) |
| Ms. Chongwan Lee | graduate research assistant (M.S) |
| Mr. Jason Papastavrou, | graduate research assistant (Ph.D) |
| *Mr. Pascal Remy | graduate research assistant (M.S) |
| Mr. Jean Louis Grevet | graduate research assistant (M.S) |

*Received M.S. degree

6. DOCUMENTATION

6.1 Theses

- [1] J. N. Tsitsiklis, "Problems in Decentralized Decision Making and Computation," Ph.D. Thesis, Dept. of EECS also Report LIDS-TH-1424, Laboratory for Information and Decision Systems, MIT, Cambridge, Mass., December 1984.
- [2] G. J. Bejjani, "Information Storage and Access in Decisionmaking Organizations," MS Thesis, Technology and Policy Program, also Report LIDS-TH-1434, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, January 1985.
- [3] V. Y.-Y. Jin, "Delays for Distributed Decisionmaking Organizations," MS Thesis, Mechanical Engineering, also Report LIDS-TH-1459, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, May 1985.
- [4] K. L. Boettcher, "A Methodology for the Analysis and Design of Human Information Processing Organizations," Ph.D. Thesis, Dept. of Electrical Engineering and Computer Sciences, also Report LIDS-TH-1501, Laboratory for Information and Decision Systems, MIT, September 1985.
- [5] R. P. Wiley, "Performance Analysis of Stochastic Timed Petri Nets," Ph.D. Thesis, Dept. of Electrical Engineering and Computer Sciences, also, Report LIDS-TH-1525, Laboratory for Information and Decision Systems, MIT, January 1986.
- [6] J. D. Papastavrou, "Distributed Detection with Selective Communications," MS Thesis, Dept. of Electrical Engineering and Computer Sciences, also Report LIDS-TH-1563, Laboratory for Information and Decision Systems, MIT, May 1986.
- [7] C. M. Bohner, "Computer Graphics for Systems Effectiveness Analysis," MS Thesis, Technology and Policy Program, also Report LIDS-TH-1573, Laboratory for Information and Decision Systems, MIT, June 1986.
- [8] H. P. Hillion, "Performance Evaluation of Decisionmaking Organizations Using Timed Petri Nets," MS Thesis, Technology and Policy Program, also Report LIDS-TH-1590, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, August 1986.
- [9] P. A. Remy, "On the Generation of Organizational Architectures Using Petri Nets," MS Thesis, Technology and Policy Program, also Report LIDS-TH-1630, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, December 1986.

6.2 Technical Papers

- [1] R. P. Wiley and R. R. Tenney, "Performance Evaluation of Stochastic Timed Decision-Free Petri Nets," Proc. 24th IEEE Conference on Decision and Control, December 11-13, 1985, Ft. Lauderdale, Florida, pp. 558-563.
- [2] G. J. Bejjani and A. H. Levis, "Information Storage and Access in Decisionmaking Organizations," LIDS-P-1466, Laboratory for Information and Decision Systems, MIT, May 1985.
- [3] J. N. Tsitsiklis and M. Athans, "On the Complexity of Decentralized Decision Making and Detection Problems," IEEE Trans. on Automatic Control, Vol. AC-30, No. 5, May 1985.
- [4] V. Y.-Y. Jin and A. H. Levis, "Computation of Delays in Acyclical Distributed Decisionmaking Organizations," LIDS-P-1488, Laboratory for Information and Decision Systems, MIT, August 1985.
- [5] K. L. Boettcher and R. R. Tenney, "A Case Study in Human Team Decisionmaking", Proc. Systems, Man, and Cybernetics Symposium, Tucson, AZ, November 1985.
- [6] K. L. Boettcher and R. R. Tenney, "On the Analysis and Design of Human Information Processing Organizations," LIDS-P-1503, Proc. 8th MIT/ONR Workshop on C³ Systems, December 1985.
- [7] K. L. Boettcher and R. R. Tenney, "Distributed Decisionmaking with Constrained Decision Makers - A Case Study," Proc. 8th MIT/ONR Workshop on C³ Systems, December 1985, also in IEEE Trans. on Systems, Man, and Cybernetics, Vol. SMC-16, No. 6, November/December 1986.
- [8] R. P. Wiley and R. R. Tenney, "Calculating Time-Related Performance Measures of a Distributed Tactical Decisionmaking Organization Using Stochastic Timed Petri Nets," Proc. 8th MIT/ONR Workshop on C³ Systems, December 1985.
- [9] V. Y.-Y. Jin, A. H. Levis, and P. A. Remy, "Delays in Acyclical Distributed Decisionmaking Organizations," LIDS-P-1528, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, also in Proc. 4th IFAC/IFORS Symposium on Large Scale Systems: Theory and Applications, Zurich, Switzerland, August 1986.
- [10] J. Papastavrou and M. Athans, "A Distributed Hypotheses-Testing Team Decision Problem with Communications Cost," LIDS-P-1538, Laboratory for Information and Decision Systems, MIT, February 1986, also in Proc. of 25th IEEE Conference on Decision and Control, December 1986.
- [11] J. Tsitsiklis, "On Optimal Thresholds in Decentralized Detection," Proc. of 25th IEEE Conference on Decision and Control, December 1986.

- [12] A. H. Levis and S. K. Andreadakis, "Computer-Aided Design of Organizations," Proc. 25th IEEE Conference on Decision and Control, Athens, Greece, December 1986.
- [13] P. Remy, A. H. Levis, V. Jin, "On the Design of Distributed Organizational Structures," LIDS-P-1581, Laboratory for Information and Decision Systems, MIT, July 1986, to appear in Proc. 10th IFAC World Congress, Munich, FRG, July 1987.
- [14] M. Athans, "Command-and-Control (C^2) Theory: A Challenge to Control Science," LIDS-P-1584, Laboratory for Information and Decision Systems, MIT, Cambridge, MA, September 1986, also accepted for publication in IEEE Trans. on Automatic Control, April 1987.
- [15] P. A. Remy and A. H. Levis, "On the Generation of Organizational Architectures Using Petri Nets," submitted to the Eighth European Workshop on Applications and Theory of Petri Nets, Zaragoza, Spain, June 24-26, 1987.
- [16] H. P. Hillion and A. H. Levis, "Timed Event-Graph and Performance Evaluation of Systems," submitted to the Eighth European Workshop on Applications and Theory of Petri Nets, Zaragoza, Spain, June 24-26, 1987.
- [17] J. N. Tsitsiklis, D. P. Bertsekas, and M. Athans, "Distributed Asynchronous Deterministic and Stochastic Gradient Optimization Algorithms," IEEE Trans. on Automatic Control, Vol. AC-31, No. 9, September 1986.

Distributed Decisionmaking with Constrained Decisionmakers: A Case Study

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Abstract—A specific distributed decisionmaking problem is considered that reflects essential aspects of tactical command and control teams, particularly communications and limitations to human information processing induced because of time pressure. A constrained optimization problem is formulated to maximize overall team performance subject to individual limitations. The results of a systematic investigation of possible solutions are described in terms of performance and workload interactions. In particular, optimal solutions arise where team members act randomly and/or introduce errors. Though results obtained are specific to the case study, a number of more generally applicable principles are evident.

I. INTRODUCTION

IN A VARIETY of tactical command and control situations it is often the case that information and/or authority are distributed among several commanders or decisionmakers, either because of geographical separation or simply because the amount of information to be processed is too great. For the overall command and control organization to function effectively relevant information must be exchanged in a timely manner so that coordinated and informed decisions are made by individual organization members. However, because individual members are subject to information processing limitations, it is necessary to select protocols and decisionmaking procedures so that the workload remains within prescribed limits. Thus a key issue in the analysis of distributed decisionmaking situations is to understand how overall team performance is affected by each individual's workload limitations.

The purpose of this paper is to examine a specific distributed decisionmaking situation in detail. Though the structure of each problem component appears simple, the analysis of the overall problem reveals that complex and subtle relationships occur, some of which give rise to guidelines for examining actual distributed command and control situations.

The case study to be considered is basically a modified team theoretic problem that is motivated by an informa-

tion processing situation typical of a command and control context. In the usual team theoretic analysis a main goal is to obtain normative decision rules that represent the desired behavior of each decision agent or team member [1]. Actual member behavior may be different than normative, however, due to unmodeled aspects of human behavior. The present formulation explicitly incorporates descriptive models of actual human behavior that represent the processing load incurred in task execution. Thus the problem is to choose a decision rule for each member so that team performance is optimized *subject to* feasibility with respect to individual workload limits.

The next section details the case study problem. A two-member tandem structure is used, and the characteristics of normative team behavior for this structure are reviewed. A key feature of team member decision rules is the presence of thresholds which each member uses to make comparison tests. A model for the information processing required to execute such a test is then described, with processing time used as the measure of workload. The complete model for each member's actual behavior includes a second element, however, which accounts for behavior when the processing time for threshold tests exceeds the time allowed. This element derives from human ability to trade accuracy for speed. Two different trade-off mechanisms are illustrated, one for each member. The overall actual behavior and processing load realized is parameterized by the decision thresholds used and by certain other parameters that figure in the speed/accuracy trade-off capability. The modified team theoretic problem is then to place these parameters for best team performance subject to the processing time used being within that available for each member. Section III discusses the characteristics of the problem solution. A particular consideration of interest is whether, and if so under what conditions, it remains desirable to retain the thresholds obtained in the original (unconstrained) team problem. Section IV investigates a special case of the problem from which principles of general interest are apparent. Finally, Section V summarizes the paper.

II. PROBLEM FORMULATION

Consider the situation where a detection decision is continually made regarding the presence or absence of a particular target, such as an enemy submarine. Suppose that two physically separated platforms make observations

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IEEE Log Number 8610741.

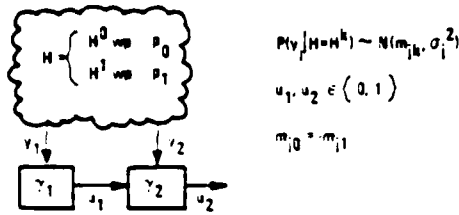


Fig. 1. Team structure.

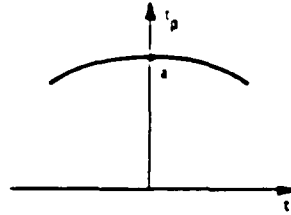


Fig. 2. Processing time for threshold comparison test.

relevant to this decision. Because of the complexities involved in analyzing observations, e.g., the interpretation of sonar data, human capabilities are used to process observations into assessments of a target's presence or absence. These assessments take time, i.e., a level of workload is induced on the individuals who are performing them. Moreover, once the assessments are made it is necessary to incorporate them into an overall decision regarding a target's presence or absence, and this process induces additional workload. In subsequent paragraphs a team structure appropriate for this scenario is suggested, and a mathematical model is developed that captures its important aspects.

Team Structure

The organization structure to be considered is illustrated in Fig. 1. Each team member receives a conditionally independent Gaussian observation on the presence or absence of a given external event H . Based on his observation, the first member selects one of two symbols to send as a coded message to the second member. The latter then incorporates his own measurement with the message to make a detection decision for the team. In the absence of any other constraints the decision rules γ_j^* that correspond to normative behavior for each team member, and which minimize the total probability of error, are known [2]:

$$\begin{aligned} \gamma_1^*: & \quad \gamma_2^*: \\ \text{if } y_1 \geq t_1^*, & \quad u_1 = 1 \quad \text{if } y_2 \geq t_2^*, \quad u_2 = 1 \\ \text{if } y_1 < t_1^*, & \quad u_1 = 0 \quad \text{if } y_2 < t_2^*, \quad u_2 = 0. \end{aligned} \quad (1)$$

The decision rules in (1) are such that

$$t_{20} > t_{21} \quad (2)$$

which means that if the first member indicates that $H = H^0$ by selecting $u_1 = 0$, the second member uses a threshold that biases him to agree with the first member's indication. A symmetric solution exists that interchanges the values of

t_{20} and t_{21} and lets the first member select $u_1 = 1$ when he wants to indicate that $H = H^0$. Though this latter possibility exhibits identical performance here, it can become an important basis for distinction when limitations in processing are introduced.

Information Processing Models

In reality, the threshold comparison test in (1) are to be accomplished by humans.¹ The observation would be displayed visually as, say, a horizontally displaced dot with the threshold displayed as a vertical line. Viewing such a display and selecting a response based on the relative positions of the dot and line takes time. Empirically, the threshold position has an effect on the time required to select a response. In particular, a comparison with a threshold t requires, on the average, t_p seconds to make where

$$t_p = t_p(t) = a - b \cdot (t)^2, \quad a > 0, \quad b \geq 0 \quad (3)$$

as shown in Fig. 2. This model captures the fact that observations are centered on zero and that response time tends to decrease as the uncertainty in the response required decreases [3]. In (3) as t becomes large in absolute value (for $b \neq 0$), most observations will fall only on one side of t , so a human can partially prejudge his response. Using the model in (3) as a basic building block, processing time descriptions for each member can now be defined.

First Team Member: The first team member performs his task using a single threshold. The processing time required to do this test is given by (3); specifically, it is denoted by $t_{p1n}(t_1) = a_1 - b_1 \cdot (t_1)^2$. In addition, it is assumed that the input/output behavior realized is such that a flawless comparison can be made (provided sufficient time is available). Denote by \bar{k}_{1n} the nominal conditional distribution $p(u_1|y_1)$ realized using the threshold test. The model is then that of

$$\bar{k}_{1n}: p(u_1 = 0|y_1) = \begin{cases} 1, & y_1 \leq t_1 \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Suppose now that the operation of the team is such that the member must complete comparison tests at the rate of one every $\bar{\tau}_1$ seconds. If it happens that t_1 is set such that $t_{p1n}(t_1) > \bar{\tau}_1$, the member will be overloaded. Therefore, an alternative processing mode is provided: an option to "guess," i.e., essentially to ignore the observation y_1 and to respond arbitrarily, choosing $u_1 = 1$ with some guessing probability g_1 . Input/output behavior with guessing is modeled by the conditional distribution \bar{k}_{1g} where

$$\bar{k}_{1g}: p(u_1 = 0|y_1) = 1 - g_1. \quad (5)$$

To make this a viable option, the time required to exercise

¹It is certainly true that the relatively straightforward processing indicated by each decision rule could be accomplished by machine. However, for purposes of illustration and for investigating the effects of workload constraints on team behavior, the model used here is a reasonable abstraction of the situation where humans are required to make judgments based on noisy observations.

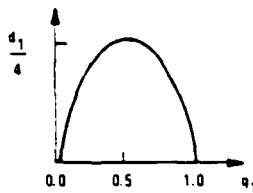


Fig. 3. Switching overhead model.

it, denoted by t_{p1g} , must be less than $t_{p1n}(t_1)$ for some range of t_1 .²

Because the team member has two options, generally, an additional amount of processing time will be required to switch between them. Switching overhead has been observed to depend on switching frequency [4]. Specifically, it can be modeled by an expression such as

$$d_1 \cdot (1 - q_1) \cdot q_1 \quad (6)$$

which is illustrated in Fig. 3. In (6), q_1 is the frequency of guessing and d_1 is a scale factor. Note that when one option is used exclusively, (6) is zero.

In sum, the actual decision rule executed by the first member is given by K_1 as follows:

$$K_1: p(u_1 = 0|y_1) = \begin{cases} (1 - q_1) + q_1 \cdot (1 - g_1), & y_1 \leq t_1 \\ q_1 \cdot (1 - g_1), & y_1 > t_1 \end{cases} \quad (7)$$

The input/output behavior in (7) has an associated average processing time of T_{p1} where

$$T_{p1} = (1 - q_1) \cdot t_{p1n}(t_1) + q_1 \cdot t_{p1g} + d_1 \cdot (1 - q_1) \cdot q_1. \quad (8)$$

The model given in (7) and (8) is basically the so-called fast-guess model [5], which reflects one mechanism whereby humans can trade speed for accuracy.

Second Team Member: The second team member switches between two thresholds using the message received from the first as a cue. Assuming an overhead for switching similar to (6), the average time required to accomplish this task depends on the threshold values and the relative frequency of using them as follows:

$$T_{p2} = \sum_{i=0}^1 \left[p(u_1 = i) \cdot (a_{2i} - b_{2i} \cdot (t_{2i})^2) \right] + d_2 \cdot p(u_1 = 0) \cdot p(u_1 = 1). \quad (9)$$

As with the first team member, the second member is subject to a processing time limit; in this case the limit takes the form of a deadline τ_2 . So long as $T_{p2} \leq \tau_2$, the team member can accomplish his processing without error. Errors will be made, however, if $p(u_1)$, t_{20} , and t_{21} are such that $T_{p2} > \tau_2$. The likelihood of errors depends on the difference between τ_2 and T_{p2} . Expressed analytically, the

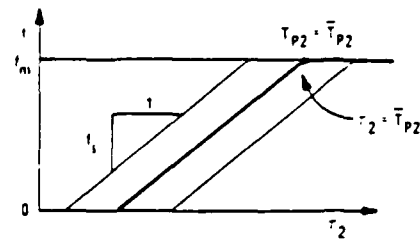


Fig. 4. Second member speed/accuracy trade-off.

input/output behavior of the second member, denoted K_2 , is as follows:

$$K_2: \text{ if } u_1 = i \quad (i = 0, 1), \text{ then}$$

$$p(u_2 = 0|y_2) = \begin{cases} q_2, & y_2 \geq t_{2i} \\ 1 - q_2, & y_2 < t_{2i} \end{cases} \quad (10)$$

where q_2 is a new parameter that represents the likelihood of threshold comparison errors. In other words, the second member performs the threshold comparison test correctly a fraction $(1 - q_2)$ of the time and makes an error on the fraction q_2 of the observations processed.

The specific relationship between input/output errors q_2 and time stress (T_{p2} versus τ_2) is based on a second mechanism whereby humans have been known to trade speed for accuracy. Pew [6] has observed a loglinear relationship between the "odds ratio" $(1 - q_2)/q_2$ and processing time. Specifically, assume that

$$q_2 = q_2(T_{p2}, \tau_2) = (1 + e^{f(T_{p2}, \tau_2)})^{-1} \quad (11)$$

and

$$f = \begin{cases} f_s \cdot (\tau_2 - T_{p2}) + f_m, & T_{p2} \geq \tau_2 \\ f_m, & T_{p2} < \tau_2 \end{cases} \quad (12)$$

For analytical convenience it is assumed that $f_m < \infty$, which effectively means that the minimum value of q_2 is nonzero.³

The relationship defined in (11) and (12) is illustrated in Fig. 4. To understand the behavior assumed for the second member, consider the special case where the thresholds t_{2i} and the distribution $p(u_1)$ are fixed, implying a value of processing time \bar{T}_{p2} for the second member. So long as \bar{T}_{p2} is less than τ_2 , the member is able to perform his processing task at maximum accuracy f_m ; in Fig. 4 this corresponds to the segment on the $f = f_m$ line. As τ_2 is decreased below \bar{T}_{p2} , however, the member has insufficient time to perform his task, so accuracy begins to suffer; the logarithm of the odds ratio declines linearly with decreasing τ_2 , where the rate of decline is given by the parameter f_s . Implicit in the model is that accuracy never declines below $f = 0$, which corresponds to completely random behavior by the second member. Finally, note that since they can be derived from

² The inclusion of a guessing option is made somewhat arbitrarily. In a more complex situation one might consider providing the member with an alternative procedure that is less detailed or less thorough. Such a procedure would save time but also would result in a response that is subject to error or is otherwise of lesser quality.

³ In (11) and (12) the deadline for the second member is represented as a free variable τ_2 . Subsequent analysis presented in this paper will, however, assume that τ_2 is fixed at a given value $\bar{\tau}_2$. If τ_2 were left as an independent variable subject to some maximum limit, it is straightforward to show that due to the monotonic relationship of q_2 and τ_2 the second member's deadline should always be set at that maximum.

errors are absent. Essentially, if the rate of accuracy degradation (σ_1) is great enough, it is desirable to operate the second member at his maximum accuracy level. In general, however, some trade-off will exist between the two extremes, and operation of the second member will fall between points C and B.

First Team Member Operation

Turning to the solution of problem CTD as it affects the first member, the key issue is to identify the circumstances under which guessing is required by the first member. A second issue is whether the solution to problem CTD can involve the use of the original normative decision threshold t_1^* . These issues can be resolved by considering in geometric terms how feasible (t_1, q_1) values map to π values.

For fixed *a priori* probabilities on H , (p_0, p_1) it is possible to characterize all π values in the (p_{00}, p_{11}) plane as t_1 and q_1 range over their possible values. Using the fact that $p_{01} = p_1 - p_{11}$ and that $p_{10} = p_0 - p_{00}$, π is completely determined by p_{00} and p_{11} . Furthermore, because the first member's overall behavior is really the combination of two distinct options, it is possible to write π in like terms as follows:

$$\pi = (1 - q_1) \cdot [p_{00r}(t_1), p_0 - p_{00r}(t_1), p_1 - p_{11r}(t_1), p_{11}]' + q_1 \cdot [(1 - g_1) \cdot p_0, g_1 \cdot p_0, (1 - g_1) \cdot p_1, g_1 \cdot p_1]' \quad (14)$$

Recall that q_1 is the fraction of guessing by the first member. The first term in brackets in (14) represents the distribution $p(u_1, H)$ corresponding the exclusive use of the threshold comparison option. It depends, as indicated, on the value of t_1 selected. The functions $p_{irr}(t_1)$ are given by

$$p_{irr}(t_1) = \Phi\left(\frac{t_1 - m_{1i}}{\sigma_1}\right) \cdot p_i, \quad i = 0, 1 \quad (15)$$

where $\Phi(\cdot)$ is the unit normal cumulative distribution function.

A region of possible π values is determined typically as shown in Fig. 7. The upper boundary of the region is the locus where $q_1 = 0$. Points Y and Z correspond to where $t_1 \rightarrow -\infty$ and $+\infty$, respectively. The lower boundary is the locus of points determined when $q_1 = 1$ and the guessing bias ranges from zero to one. Point S corresponds to 50/50 guessing probabilities, i.e., to where $g_1 = 0.5$. When viewed as part of the lower boundary, points Y and Z correspond to $g_1 = 1$ and 0, respectively. In terms of the underlying (t_1, q_1) values, any point in the interior or on the diagonal boundary represents a nonzero guessing fraction by the team member. Note that the normative threshold value of t_1 is, therefore, on the upper boundary as illustrated. The geometric representation in Fig. 7 has many properties in common with the receiver operating characteristic in signal detection theory [7]. In particular, better team performance generally results when the operating point in the (p_{00}, p_{11}) plane moves nearer to (p_0, p_1) .

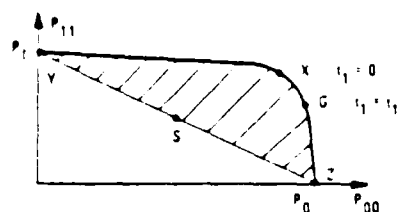


Fig. 7. Typical region of π values in (p_{00}, p_{11}) plane.

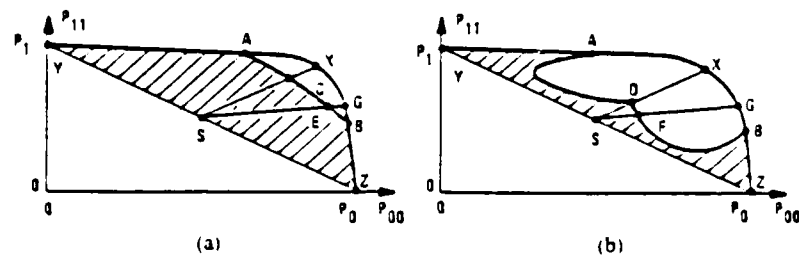
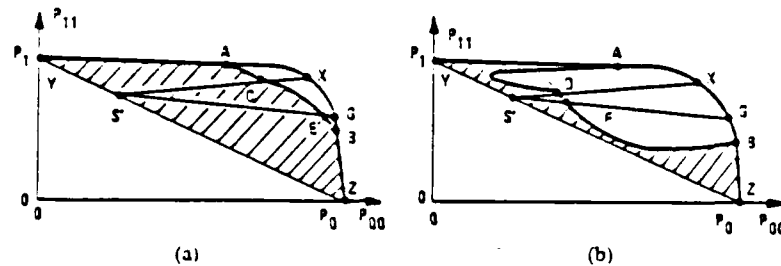
Consider now the solution of problem CTD with respect to the first member. Since a one-to-one correspondence exists between (q_1, t_1) values and (p_{00}, p_{11}) values, possible solutions can be examined directly in terms of (p_{00}, p_{11}) points. While Fig. 7 represents possible π values, not all of them will be feasible due to the constraint on T_{p1} . Fig. 8(a) shows typically how this constraint restricts π values for $d_1 = 0$, i.e., when the first member has no switching overhead. (A guessing bias of 0.5 has been assumed.) The arc ACB represents the locus where $T_{p1} = \bar{\tau}_1$, and the shaded area designates the region of feasible π values. A similar depiction is given in Fig. 8(b) except that d_1 has increased from zero to a relatively significant value. Here the arc ADB represents the locus where $T_{p1} = \bar{\tau}_1$.

The solution for (q_1, t_1) is found by searching over regions such as those in Fig. 8. It can be shown, however, (see the Appendix) that a solution to problem CTD is such that either $q_1 = 0$ or $T_{p1} = \bar{\tau}_1$. This corresponds to the upper boundary of the feasible region. In terms of Fig. 8(a) and (b) the solution must be on the arcs YACBZ or YADBZ, respectively. In particular, it is possible that solutions will be obtained on the arcs ACB or ADB, which means that *it may be optimal to guess*.

This can be explained qualitatively by noting that it is desired to operate in the (p_{00}, p_{11}) plane as close as possible to the point where $q_1 = 0$ and $t_1 = t_1^*$ (point G). In Fig. 8 neither region admits the exclusive use of t_1^* . In Fig. 8(a), however, point E is closer than point B, where the former is such that $q_1 \neq 0$ and the latter is the nearest feasible point where $q_1 = 0$. In Fig. 8(b) point B is closer to the normative solution point. Thus the solution in Fig. 8(a) is likely to have a solution where $q_1 \neq 0$, while in Fig. 8(b) the solution will likely be at point B. Though shown for cases where $d_1 = 0$ or $d_1 \neq 0$ this behavior does not represent a special case tied to the presence of switching overhead, nor is it dependent on having the bias in guessing at 0.5. Fig. 9 shows the same constraints for a bias of $g_1 = 0.75$.

Thus as with the second member, for purposes of optimizing team performance it may be desirable to have the first member behave randomly a fraction of the time.⁵ Though this may seem counterintuitive it is a direct consequence of the interaction between performance and workload. Furthermore, the result emphasizes the fact that workload and performance, though dependent on the same fundamental parameters, are really distinct quantities. This

⁵ Contrast this with a basic result of normative multiperson team theory which states that a deterministic strategy always exists whose performance is no worse than any given randomized strategy.

Fig. 8. Constraint on T_{p1} in (p_{00}, p_{11}) plane; $g_1 = 0.5$. (a) $J_1 = 0$. (b) $J_1 = J$.Fig. 9. Constraint on T_{p1} in (p_{00}, p_{11}) plane; $g_1 = 0.75$. (a) $J_1 = 0$. (b) $J_1 = J$.

is particularly evident when comparing the region of possible values for π in the (p_{00}, p_{11}) plane with the region of feasible values. While the former is always convex, the latter can be decidedly nonconvex.

Reversing Signals—An Alternate Solution Concept

The characteristics of the solution of problem CTD have illustrated how team performance can be improved by carefully adjusting the workload of individual members. The trade-off between workload and performance can be a subtle one, even in the particular case study at hand. As an extreme case in point, it is possible, because of the effects of processing time constraints, that a reversal of the interpretation of the signal sent from the first member to the second can yield a lower team detection error. This phenomenon is documented in [8]. An intuitive discussion of why this can be the case follows.

Recall that in solving for normative threshold values two equivalent solutions exist. One is that the first member indicates $H = H^1$ by selecting $u_1 = 1$ if $y_1 \geq t_1^*$, which in turn selects a threshold t_{21}^* that biases the second member. A symmetric solution is for the first member to send $u_1 = 0$ when $y_1 \geq t_1^*$. However, the second member must then reverse his interpretation of the value of u_1 received when he selects his threshold t_{21}^* . Table I summarizes the two possibilities. So long as processing time constraints are not a factor, both solution types are completely equivalent.

Suppose the second member is constrained in terms of processing resources according to the model in (9). Assuming that the first member's operating characteristics are fixed, the processing time required by the second member depends only on the values of t_{21}^* . If $b_{20} \neq b_{21}$, then it is entirely possible that the two solutions in Table I may have unequal processing time requirements. In particular, assume that

$$T_{p2}^{II} < \bar{\tau}_2 < T_{p2}^I \quad (16)$$

TABLE I
EQUIVALENT NORMATIVE SOLUTIONS

| Case | Solution | Indication when $y_1 \geq t_1^*$ | Relationship of t_{21}^* |
|------|----------|----------------------------------|----------------------------|
| I | regular | $u_1 = 1$ | $t_{21}^* > t_{20}^*$ |
| II | reversed | $u_1 = 0$ | $t_{21}^* < t_{20}^*$ |

From (16) it is evident that if the regular normative thresholds (I) are implemented, the second member will be forced to trade accuracy for speed since his processing time requirements exceed the time allowed ($\bar{\tau}_2$). This is not the case, on the other hand, for implementation of the "reversed" solution. In fact, this solution would be superior to any other; that is, reversing signal interpretation can be desirable when processing limitations must be taken into account.

IV. SPECIAL CASE

To highlight particular mechanisms of how one member can affect the other and also team performance, consider the following special case. Suppose that the second member's processing time is independent of the threshold positions, i.e., $b_{21} = 0$ and that the switching overhead for the second member is significant. In addition, assume the deadline $\bar{\tau}_2$ is such that

$$a_{20} > \bar{\tau}_2 > a_{21} \quad (17)$$

which means that it takes longer to use threshold t_{20}^* than it does to use t_{21}^* . Finally, assume that the first member is workload unconstrained. For this special case problem CTD can be summarized in terms of Fig. 10. Since T_{p2} is independent of t_{21}^* , its variation is due entirely to variation in $p(u_1)$ which is determined by the first team member through placement of t_1 . The dependence of T_{p2} on $p(u_1 = 0) \triangleq q_1$ is shown in the upper part of Fig. 10. The relationship between T_{p2} and input/output errors q_2 (through f) is shown in the lower part of the figure.

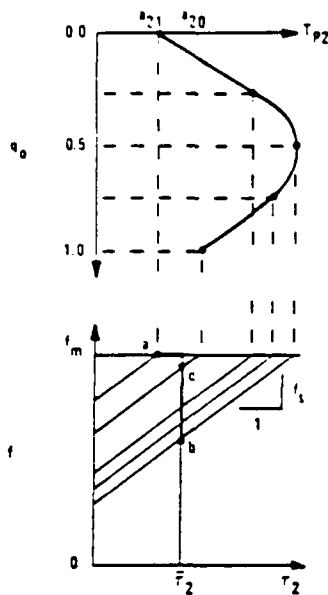


Fig. 10. Illustration of special case solution.

Recall (from (12)) that a given value of T_{p2} determines a locus of f values as a function of τ_2 . With τ_2 fixed at $\bar{\tau}_2$ a specific operating point on this locus is selected. In addition, as q_0 moves from zero to one, the resulting T_{p2} values trace out feasible operating points in the lower part of the figure, moving from a to b and back to c. The overall solution thus becomes a matter of searching over τ_1 values (and, consequently, over q_0 values) for the minimum error probability.

The interesting feature of the minimization in this special case is that the trade-off between speed and accuracy exhibited by the second member is governed entirely by the first member. Furthermore, a reduction in T_{p2} depends mostly on reducing the switching frequency. If τ_1^* is somewhere near zero, then $q_0^* \approx 0.5$ and the optimization problem is essentially one that must weigh two alternatives: either degrade the first member's quality of processing by adjusting τ_1 to reduce the switching load of the second and thereby reduce q_2 , or accept the higher input/output error rate of the second member in favor of retaining a higher quality of processing by the first.

Once the solution is obtained the thresholds will be set at the solution values and the team will presumably operate as modeled. By way of further illustrating how processing load and performance can interrelate, suppose that after the team has been set into operation the constraint on the first member becomes binding, say due to external factors that reduce the value of $\bar{\tau}_1$. As per design, the team member can resort to guessing to meet the constraint. Fig. 11 shows a trajectory in the (p_{00}, p_{11}) plane corresponding to increasing q_1 for each of two guessing biases g_1 . Point H corresponds to the problem CTD solution operating point. Points S and P correspond to completely random operation with guessing probabilities of 0.5 and 1.0, respectively. The locus of $q_0 = 0.5$ has also been shown.

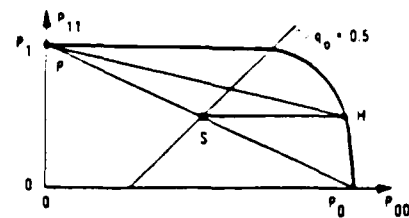


Fig. 11. Illustration of special case operation.

As q_1 increases, the operating point moves away from H to either S or P. Because the movement is toward the diagonal "guessing" line, team performance will generally be worse. However, a significant qualitative difference is apparent. Along the trajectory HS, T_{p2} is increasing and, in fact, comes to rest where switching frequency is at its maximum. This in turn implies that the degradation in accuracy (q_2) is at a maximum, which adversely affects team performance. Along the trajectory HP, however, T_{p2} first rises due to the increase in switching but decreases as switching overhead goes to zero. In this latter case the contribution to performance degradation due to input/output errors is less. These two scenarios illustrate instances of increasing processing load and degrading performance as well as decreasing load and degrading performance. Furthermore, even though operation at points P and S in Fig. 11 correspond to cases where no useful information is being passed to the second member, a significant difference exists in the processing load induced by the first on the second. Thus even in this relatively simple case it is evident that increasing workload does not necessarily lead to improved performance.

V. SUMMARY

This paper has added human processing time models and constraints to a simple team theoretic problem. These constraints significantly modify the solution characteristics. In particular, partially random behavior by team members can be optimal either through the deliberate selection of an option to guess to relieve time pressure or through selection of thresholds that require more time than is available, which in turn induces processing errors.

From the results obtained in this case study several guidelines of general interest are indicated. First, because of the variety of relationships possible between team performance and individual workload, a significant step toward understanding a given team structure is to identify which types are actually present in a given practical situation. Since in the present case study even simple models of individuals have led to complex organization behavior, it would appear that such an understanding is almost a prerequisite for successful team design. Second, the effects of switching as seen in the special case suggest a principle of general interest. Given that switching among tasks may require additional information processing resources at one location and that the amount of switching may be governed by a member at another location, the recognition of the potential for switching within a team structure may lead to

a better understanding of a specific mechanism that can result in subtle interactions and complex team behavior.

More generally, the work addresses a significant open question in the design of command and control systems. On the one hand, a scientific approach to command and control demands the application of modern quantitative analytic techniques to evaluate designs and to permit objective trade-off studies. On the other hand, command and control is clearly human intensive: man dominates machines in all deployed command and control systems, and this is unlikely to change in the near future. A critical question then is how to develop quantitative approaches to systems as large as those found in command and control when such systems possess additional complexity by having many humans embedded at critical nodes. This question has not been answered in general by this paper. However, the paper has shown that at least for human tasks that are not highly cognitive in nature, a mathematical framework can be established that incorporates both human and engineering attributes of a system. To the extent that the ideas presented here can be generalized and applied to real problems, command and control system engineers will have a method for designing and evaluating human-machine architectures that is built on objective quantitative foundations.

APPENDIX

To derive the results discussed in the paper, it is useful to reformulate problem CTD. Define

$$J(\pi, t_{20}, t_{21}) = p_{00} \cdot \left[1 - \Phi \left(\frac{t_{20} - m_{20}}{\sigma_2} \right) \right] + p_{01} \cdot \left[\Phi \left(\frac{t_{20} - m_{21}}{\sigma_2} \right) \right] + p_{10} \cdot \left[1 - \Phi \left(\frac{t_{21} - m_{20}}{\sigma_2} \right) \right] + p_{11} \cdot \left[\Phi \left(\frac{t_{21} - m_{21}}{\sigma_2} \right) \right] \quad (18)$$

Equation (18) represents the detection error probability of the team as a function of π , t_{20} , and t_{21} , assuming $q_2 = 0$. Rewriting J_2 using J and showing the decomposition by stages, problem CTD becomes

$$\min_{t_1, t_2} \left\{ \min_{t_{20}, t_{21}} \left\{ [1 - 2 \cdot J(\pi, t_{20}, t_{21})] \cdot q_2(\bar{T}_{p2}, \bar{t}_2) + J(\pi, t_{20}, t_{21}) \right\} \right. \\ \left. \text{s.t. } T_{p1} \leq \bar{t}_1 \right. \\ \left. \pi = \pi(t_1, q_1) \right\}$$

where $\pi(t_1, q_1)$ is defined by (14).

Before proceeding to an analysis of solution characteristics, it is convenient to formulate a modified version of problem CTD. Since explicit dependence on thresholds t_1 and t_{21} occurs only in the function J and in the de-

termination of processing time T_{p2} , it is possible to aggregate these thresholds into the single variable \bar{T}_{p2} and to substitute a new function \bar{J} for J , where

$$\bar{J}(\pi, \bar{T}_{p2}) = \min_{t_{20}, t_{21}} J(\pi, t_{20}, t_{21}) \quad (19) \\ \text{s.t. } T_{p2} = \bar{T}_{p2}.$$

In other words, given a π and \bar{T}_{p2} value the relationship of t_{20} and t_{21} is defined (in fact they describe an ellipse). The minimization in (19) generates threshold values \bar{t}_2 that represent the solution as a function of \bar{T}_{p2} . Furthermore, it can be assumed that the resulting \bar{J} is better than chance behavior, i.e., that $\bar{J} < \min(p_0, p_1) \leq 0.5$, which will be true in all nondegenerate cases of problem CTD. Using this aggregation, problem CTD can be stated in terms of q_1 , t_1 , and \bar{T}_{p2} as problem CTD-R.

Problem CTD-R: The problem is as follows:

$$\min_{q_1, t_1} \left\{ \min_{\bar{T}_{p2}} \left\{ [1 - 2 \cdot \bar{J}(\pi, \bar{T}_{p2})] \cdot q_2(\bar{T}_{p2}, \bar{t}_2) + \bar{J}(\pi, \bar{T}_{p2}) \right\} \right. \\ \left. \text{s.t. } T_{p1} \leq \bar{t}_1 \right. \\ \left. \pi = \pi(t_1, q_1) \right\}$$

Second Member Solution Characteristics

Using Normative Thresholds: Whereas the minimization in (19) resulted in the construction of two functions $\bar{t}_2(\bar{T}_{p2}, \pi)$, performing the minimization of J without the constraint in (19) results in two different functions that represent the normative values of t_2 for a given π value. (Included in this set is the pair of thresholds that define γ_2^* .) Denote these normative thresholds by $t_{p2}^*(\pi)$, and denote by $T_{p2}^*(\pi)$ the processing time required by the second member when they are used.

In problem CTD-R the first stage minimization is that of finding a value of \bar{T}_{p2} that solves

$$\frac{\partial \bar{J}}{\partial \bar{T}_{p2}} \cdot [1 - 2 \cdot q_2] + \frac{\partial q_2}{\partial \bar{T}_{p2}} \cdot [1 - 2 \cdot \bar{J}] = 0. \quad (20)$$

The issue at hand is whether $T_{p2}^*(\pi)$ satisfies (20). Because $\bar{T}_{p2}^*(\pi)$ represents a global minimum of \bar{J} , the first term in (20) is zero. Now if $T_{p2}^*(\pi) \leq \bar{t}_2$ the second term is also zero since q_2 does not depend on \bar{T}_{p2} in this region. Thus normative thresholds are solutions only when the processing time they require does not exceed the deadline. This is reasonable since any adjustment of thresholds would have no effect on input/output errors.

However, $T_{p2}^*(\pi) > \bar{t}_2$ leads to a different result. In this situation q_2 is monotonically increasing with \bar{T}_{p2} . Furthermore, since $\bar{J} < 0.5$ as discussed earlier, it is true that the second term is nonzero, and hence $T_{p2}^*(\pi)$ does not solve (20). This result means that if the processing time required by use of the normative thresholds is greater than that allowed, it is always desirable to adjust t_{20} and t_{21} to reduce T_{p2} and thereby reduce the second member's input/output accuracy (as reflected in the parameter q_2).

Minimizing Second Member Input/Output Errors: Given the foregoing result, the question arises as to whether

input/output accuracy should be maximized, even at the expense of the threshold settings and their impact on detection performance. In terms of problem CTD-R this issue is one of whether $\bar{T}_{p_2} = \bar{\tau}_2$ is a solution to the inner minimization, given that $\bar{T}_{p_2}^*(\pi) > \bar{\tau}_2$, or whether $\bar{T}_{p_2} > \bar{\tau}_2$ is a solution instead. Its resolution depends on how drastically the trade of speed for accuracy is made by the team member which is modeled by the parameter f_s .

To investigate this issue properly, add another constraint to problem CTD-R in the inner stage that restricts values of \bar{T}_{p_2} with respect to $\bar{\tau}_2$. The result is the problem

$$\min_{\bar{T}_{p_2}} \bar{J}(\pi, \bar{T}_{p_2}) - [1 - 2 \cdot \bar{J}(\pi, \bar{T}_{p_2})] \cdot q_2(\bar{T}_{p_2}, \bar{\tau}_2) \quad (21)$$

s.t. $\bar{\tau}_2 \leq \bar{T}_{p_2}$

where it is assumed that $\bar{T}_{p_2}^*(\pi) > \bar{\tau}_2$. The necessary conditions for a solution value of \bar{T}_{p_2} are

$$\frac{\partial \bar{J}}{\partial \bar{T}_{p_2}} \cdot [1 - 2 \cdot q_2] + \frac{\partial q_2}{\partial \bar{T}_{p_2}} \cdot [1 - 2 \cdot \bar{J}] - \mu = 0 \quad (22a)$$

$$\mu \cdot (\bar{\tau}_2 - \bar{T}_{p_2}) = 0 \quad (22b)$$

$$\mu \geq 0, \quad (22c)$$

and the issue is whether $\bar{T}_{p_2} = \bar{\tau}_2$ is a solution to (22). At $\bar{T}_{p_2} = \bar{\tau}_2$, q_2 is at its minimum: $q_2 = (1 + \exp(f_m))^{-1} \triangleq q_{2m}$. Furthermore,

$$\frac{\partial q_2(\bar{\tau}_2, \bar{\tau}_2)}{\partial \bar{T}_{p_2}} = f_s \cdot e^{-f_m} \cdot (q_{2m})^{-2}. \quad (23)$$

Substituting (23) into (22a) and rearranging gives

$$f_s > -(q_{2m})^2 \cdot e^{-f_m} \cdot \left[\frac{1 - q_{2m}}{1 - \bar{J}(\pi, \bar{\tau}_2)} \right] \cdot \frac{\partial \bar{J}(\pi, \bar{\tau}_2)}{\partial \bar{T}_{p_2}} \triangleq F_s \quad (24)$$

where F_s is a nonnegative quantity. The relationship in (23) must be satisfied if $\bar{T}_{p_2} = \bar{\tau}_2$ is a solution to (22). The parameter f_s models the rate at which input/output errors increase as the processing time required increases beyond the deadline. If $f_s > F_s$, then the marginal increase in q_2 is great enough such that it is optimal to minimize input/output errors and to adjust thresholds accordingly. If $f_s < F_s$, then a compromise exists between the two extremes—minimum q_2 at \bar{T}_{p_2} or minimum \bar{J} at $\bar{T}_{p_2} = \bar{T}_{p_2}^*$ —that gives better overall team performance.

First Member Solution Characteristics

As indicated earlier, the solution to problem CTD is found by searching over regions such as those in Fig. 7. It is not necessary to consider every feasible (p_{00}, p_{11}) point, however. Consider again the region in the (p_{00}, p_{11}) plane that represents possible $p(u_1, H)$ values. Denote this region by R and also define q_0 to be the quantity $p(u_1 = 0)$. In terms of π ,

$$q_0 = p(u_1 = 0) = p_{u_1} + p_{H_1} = p_{00} + p_1 - p_{11}. \quad (25)$$

In the (p_{00}, p_{11}) plane, constant q_0 contours are lines with positive slope as shown in Fig. 12. For each q_0 value two

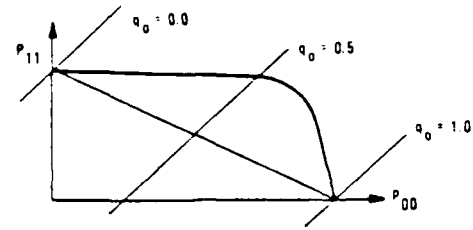


Fig. 12. Constant q_0 contours in (p_{00}, p_{11}) plane.

(p_{00}, p_{11}) pairs exist on the boundary of R . Denote the pair on the lower diagonal boundary by $(p_{00g}(q_0), p_{11g}(q_0))$ and the pair on the upper right boundary by $(p_{00u}(q_0), p_{11u}(q_0))$. Then all possible (p_{00}, p_{11}) values in R can be determined from values of q_0 and δ using the expression

$$\begin{bmatrix} p_{00} \\ p_{11} \end{bmatrix} = (1 - \delta) \cdot \begin{bmatrix} p_{00g}(q_0) \\ p_{11g}(q_0) \end{bmatrix} + \delta \cdot \begin{bmatrix} p_{00u}(q_0) \\ p_{11u}(q_0) \end{bmatrix}, \quad \delta \in [0, 1]. \quad (26)$$

Conversely, only values in R can be reached by (26). Consequently, it is possible to search over (q_0, δ) pairs to accomplish the outer minimization in problem CTD.

To set up the general solution in terms of (q_0, δ) pairs, consider first a special case of problem CTD where the constraint on the first member is not binding and all possible π values are feasible. In terms of q_0 and δ problem CTD can be written as

$$\min_{q_0, \delta} \left\{ \min_{t_{20}, t_{21}} \{ q_2 + (1 - 2q_2) \cdot J \} \right\}. \quad (27)$$

However, an equivalent problem is obtained by rearranging the order of minimization:

$$\min_{q_0} \left\{ \min_{t_{20}, t_{21}} \left\{ \min_{\delta} \{ q_2 + (1 - 2q_2) \cdot J \} \right\} \right\}. \quad (28)$$

The advantage of doing so is that for given t_{20} , t_{21} , and q_0 the value of q_2 is fixed. This means that minimization over δ affects only J . As a shorthand, define

$$\Phi \left(\frac{t_{2i} - m_{2k}}{\sigma_2} \right) \triangleq \Phi_{ik}, \quad i, k \in \{0, 1\}. \quad (29)$$

Substituting (29) into (18) and rewriting in terms of (q_0, δ) , J can be expressed as

$$\begin{aligned} J(\pi, t_{20}, t_{21}) = & [(1 - \delta) \cdot p_{00u}(q_0) + \delta p_{00g}(q_0)] \\ & \cdot (\Phi_{10} - \Phi_{00}) + p_0 \cdot (1 - q_0) \\ & + [(1 - \delta) \cdot p_{11u}(q_0) + \delta \cdot p_{11g}(q_0)] \\ & \cdot (\Phi_{11} - \Phi_{01}) + p_1 \cdot q_0. \end{aligned} \quad (30)$$

For given values of t_{20} , t_{21} , and q_0 consider the minimization over δ in (28), which is simply the minimization of J with respect to δ . Therefore, differentiate (30) with respect to δ . The result yields

$$\begin{aligned} \frac{\partial J}{\partial \delta} = & [p_{00g}(q_0) - p_{00u}(q_0)] \cdot (\Phi_{10} - \Phi_{00}) \\ & + [p_{11g}(q_0) - p_{11u}(q_0)] \cdot (\Phi_{11} - \Phi_{01}). \end{aligned} \quad (31)$$

Because of the parameterization in terms of q_0 and the properties of R it is true that

$$p_{00}(q_0) \leq p_{00}(q_0) \quad (32)$$

$$p_{11}(q_0) \leq p_{11}(q_0). \quad (33)$$

Equality in (32) and (33) only occurs at the extremes where $q_0 = 0$ or 1. Since these two situations are not of particular interest with respect to the subsequent conclusions, a strict inequality can be assumed in (32) and (33). Now, if $t_{21} < t_{20}$, then strict inequalities exist between Φ_{11} and Φ_{10} :

$$\Phi_{10} < \Phi_{00} \quad (34)$$

$$\Phi_{11} < \Phi_{01}. \quad (35)$$

In this situation the right-hand side of (31) is strictly positive, which means that J increases with increasing δ . Thus the minimizing δ is zero, which means that the solution is on the upper boundary of R . A complementary situation obtains for $t_{20} < t_{21}$: $\delta = 1$ and the solution is on the lower boundary. For the case where $t_{20} = t_{21}$ any value of δ is a solution; $\delta = 0$ is arbitrarily specified.

Since the observations made with respect to (31) are valid for any q_0 , the general conclusion is that solutions to problem CTD are such that they fall on the boundary of R . Furthermore, it can be argued on intuitive grounds that the solution must be on the upper boundary. This is reasoned as follows. For given q_0 , knowing that $\delta = 0$ or 1 in effect reduces the problem in (28) to two minimizations over t_{20}, t_{21} :

$$\min_{t_{20}, t_{21}} q_2 + (1 - 2q_2) \cdot J_u(q_0) \quad (36)$$

$$\min_{t_{20}, t_{21}} q_2 + (1 - 2q_2) \cdot J_s(q_0) \quad (37)$$

where J_u and J_s represent the values of J on the upper and lower boundaries of R , respectively. By analogy with the receiver operating characteristic, however, the lower boundary represents purely random responses by the first member. Furthermore, because since q_0 is given, the effect on q_2 by the first member is the same in (36) and (37). Finally, no restrictions are on t_{21} in either case. Given these facts, the issue is whether the solution in (36) yields a smaller J_0 than in (37). Since (36) represents operation at a point where the first member is providing *some* useful indication to the second member, it can be concluded that the team can do no worse in (36) than in (37) because in the latter case *no* useful information is provided by the first member.

The foregoing discussion has been made for the special case where the entire region of realizable (p_{00}, p_{11}) pairs was also feasible. If the processing time constraint on the first member is binding, then not all of R is feasible as illustrated in Figs. 8 and 9. The parameterization of (p_{00}, p_{11}) pairs in terms of q_0 and δ must be adjusted in this case so that only feasible (p_{00}, p_{11}) pairs are obtained. This can be done simply by restricting δ . That is, for each q_0 value a set of δ values that correspond to feasible (p_{00}, p_{11}) pairs will exist. Denote this set by $\Delta(q_0)$. Thus

the problem in (27) is modified to

$$\min_{q_0, \delta} \left\{ \min_{t_{20}, t_{21}} \{ q_2 + (1 - 2q_2) \cdot J \} \right\} \quad (38)$$

$$\text{s.t. } \delta \in \Delta(q_0)$$

The arguments made earlier with respect to $\partial J / \partial \delta$ are unchanged, however. It is still desirable to place δ at either its maximum or minimum value. This means that solutions to (38) will either be on the lower diagonal or on the upper boundary of the region representing feasible (p_{00}, p_{11}) values. Furthermore, the same line of reasoning can be used to argue that the upper boundary represents a uniformly better solution point for a given q_0 . Taking this to be the case, the general conclusion is that solution to problem CTD is such that either

$$q_1 = 0 \quad \text{or} \quad T_{p1} = \bar{T}_1. \quad (39)$$

REFERENCES

- [1] Y. C. Ho, "Team decision theory and information structures," *Proc. IEEE*, vol. 68, pp. 664-654, June 1980.
- [2] L. K. Ekelman and R. R. Tenney, "Detection networks," in *Proc. 21st IEEE Conf. Decision and Control*, 1982, pp. 686-691.
- [3] T. B. Sheridan and W. R. Ferrell, *Man-Machine Systems*. Cambridge, MA: MIT Press, 1974.
- [4] K. L. Boettcher and R. R. Tenney, "A procedure-based approach to human information processing models," Lab. Inform. Decision Syst., Mass. Inst. Technol., Cambridge, Rep LIDS-P-1499, 1985.
- [5] J. I. Yellot, "Correction for fast guessing and the speed-accuracy tradeoff in choice reaction time," *J. Math. Psych.*, vol. 8, pp. 159-199, 1971.
- [6] R. W. Pew, "The speed-accuracy tradeoff," *Acta. Psych.*, vol. 30, pp. 16-26, 1969.
- [7] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Vol. I*. New York: Wiley, 1968.
- [8] K. L. Boettcher, "A methodology for the analysis and design of human information processing organizations," Lab. Inform. Decision Syst., Mass. Inst. Technol., Cambridge, Rep. LIDS-T-1501, 1985.
- [9] D. G. Luenberger, *Introduction to Linear and Nonlinear Programming*. Reading, MA: Addison-Wesley, 1973.



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